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On the use of a running coupling in the calculation of forward hadron production at next-to-leading order

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Abstract

We study a puzzle raised recently regarding the running coupling prescription used in the calculation of forward particle production in proton-nucleus collisions at next-to-leading order: using a coordinate space prescription which is consistent with the one used in the high energy evolution of the target leads to results which can be two orders of magnitude larger than the ones obtained with a momentum space prescription. We show that this is an artefact of the Fourier transform involved when passing between coordinate and momentum space and propose a new coordinate space prescription which avoids this problem.

Keywords: Color Glass Condensate, Saturation

1. Introduction

Forward particle production in high energy proton-proton or proton-nucleus collisions is an important probe of the nuclear wavefunction at small x, where nonlinear effects such as gluon saturation are expected to become sizable. In the past few years, the Color Glass Condensate (CGC) effective theory, which is the natural framework to study such processes, was promoted to next-to-leading order (NLO) accuracy, as required to improve the predictability of this formalism. This includes the NLO corrections both to the Balitsky-Kovchegov (BK) evolution [1, 2] of gluon densities in a dense nuclear target and to the hard part describing the coupling of a dilute projectile with this target. Unfortunately, the first numerical studies implementing these corrections met with unphysical results such as instability of the NLO high energy evolution [3] and negativity of the forward particle production cross-section [4]. In the latter case, the problem appears in the range of semi-hard transverse momenta where the CGC formalism is supposed to be applicable. As shown in [5], this issue is related to the way one separates the target evolution from the impact factor. In [6], a reformulation of the cross-section was proposed, which leads to positive results at all transverse momenta at fixed coupling as demonstrated explicitly in [7] (a similar observation was

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recently made in the calculation of the DIS structure functions at NLO [8]). However, the correct way of implementing the running of the coupling is still an issue. Indeed, the BK equation is most naturally solved in coordinate space, while the cross-section is written in momentum space. In [7] it was shown that a mixed treatment, where different prescriptions for the coupling are used in the impact factor and when solving the BK equation, can make the negativity issue appear again. On the other hand, it was found that the use of the same coordinate space prescription in the whole calculation leads to another problem, with the NLO result becoming orders of magnitude larger than the LO one at large transverse momenta. Our main goal here is thus to understand the origin of this puzzle and to identify a running coupling prescription that can lead to physical results.

2. Results

For simplicity, we focus here on the $q \to q$ channel and do not consider the fragmentation functions. The LO quark multiplicity reads

$$\frac{\mathrm{d}N^{\mathrm{LO}}}{\mathrm{d}^2\boldsymbol{k}\,\mathrm{d}\eta} = \frac{x_p q(x_p)}{(2\pi)^2}\,\mathcal{S}(\boldsymbol{k}, X_g)\,, \tag{1}$$

where $\boldsymbol{k}$ and η are the transverse momentum and rapidity of the produced quark respectively, $x_p = (k_\perp e^{\eta})/\sqrt{s}$, $X_g = (k_\perp e^{-\eta})/\sqrt{s}$, and $q(x)$ is the quark distribution in the projectile proton. $\mathcal{S}$ is the Fourier transform of the dipole correlator in the color field of the target,

$$\mathcal{S}(\boldsymbol{k}, X) = \int \mathrm{d}^2\boldsymbol{r}\,\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}} S(\boldsymbol{r}, X)\,, \quad S(\boldsymbol{x}, \boldsymbol{y}; X) = \frac{1}{N_c}\left\langle \mathrm{tr}\left[V(\boldsymbol{x})V^{\dagger}(\boldsymbol{y})\right]\right\rangle_X\,, \tag{2}$$

and its evolution as a function of X obeys the Balitsky-Kovchegov equation. The LO multiplicity (1) receives NLO corrections associated with the emission of a hard primary gluon which can be divided into two classes, depending upon the overall color factor: N_c or C_F [9, 10]. We first consider the N_c terms which were identified in [5] as the origin of the negativity problem observed in [4]. The sum of the LO and N_c NLO contributions reads, in the "unsubtracted" form [6],

$$\begin{aligned}\frac{\mathrm{d}N^{\mathrm{LO}+N_c}}{\mathrm{d}^2\boldsymbol{k}\,\mathrm{d}\eta} = {}& \frac{x_p q(x_p)}{(2\pi)^2}\mathcal{S}(\boldsymbol{k}, X_0) + \frac{1}{4\pi}\int_0^{1-X_g/X_0}\mathrm{d}\xi\,\frac{1+\xi^2}{1-\xi} \\ & \times\left[\Theta(\xi - x_p)\frac{x_p}{\xi}\,q\left(\frac{x_p}{\xi}\right)\mathcal{J}(\boldsymbol{k}, \xi, X(\xi)) - x_p q(x_p)\mathcal{J}_v(\boldsymbol{k}, \xi, X(\xi))\right],\end{aligned} \tag{3}$$

where X_0 corresponds to the initial condition for the BK evolution of the target. The functions $\mathcal{J}$ and $\mathcal{J}_v$ can be written as Fourier transforms of coordinate-space integrals:

$$\mathcal{J}(\boldsymbol{k}, \xi, X(\xi)) = \int \mathrm{d}^2\boldsymbol{r}\,\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}} J(\boldsymbol{r}, \xi, X(\xi))\,, \quad \mathcal{J}_v(\boldsymbol{k}, \xi, X(\xi)) = \int \mathrm{d}^2\boldsymbol{r}\,\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}} J_v(\boldsymbol{r}, \xi, X(\xi))\,, \tag{4}$$

where the expressions for J and J_v can be found in [11]. When the transverse momentum $k_\perp$ of the produced particle is significantly larger than the target's saturation scale Q_s, this $k_\perp$ cannot be taken from the target via multiple scattering and hence it must be balanced by the unobserved gluon. In coordinate space, this means that the dominant contribution to the cross-section must come from the region $x_\perp \sim r_\perp$, where $\boldsymbol{x}$ is the transverse coordinate of the primary gluon. This physical condition is satisfied if one considers a fixed coupling or the momentum-space running coupling $\alpha_s(k_\perp)$, but not also for a running coupling, like $\alpha_s(r_\perp)$, which depends upon the parent dipole size $r_\perp$. To see this, consider the contribution from the complementary region at $x_\perp \gg r_\perp$, which on physical grounds is expected to be unimportant. This can be estimated as [11]

$$\mathcal{J}(\boldsymbol{k}, \xi) \sim \int \mathrm{d}^2\boldsymbol{r}\,\frac{\bar{\alpha}_s}{2\pi^2}\,\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\int_{r_\perp}\frac{\mathrm{d}^2\boldsymbol{x}}{\boldsymbol{x}^2}\,\left[S((1-\xi)\boldsymbol{x}) - S(-\xi\boldsymbol{x})S(\boldsymbol{x})\right] \quad \text{for } x_\perp \gg r_\perp\,, \tag{5}$$

and similarly for $\mathcal{J}_v$. The combination of dipole S-matrices within the square brackets grows like $x_\perp^2$ for small $x_\perp \sim r_\perp$, while it exponentially vanishes for larger $x_\perp \gtrsim 1/Q_s$. Accordingly the integral over $\boldsymbol{x}$ is

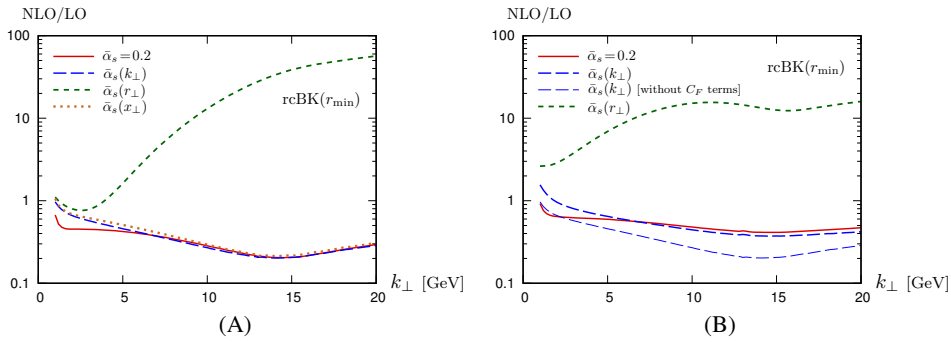

Fig. 1. Left: Ratio of the NLO multiplicity (including only the N_c terms) and the LO one for different prescriptions of the running coupling. Right: Ratio of the total NLO quark multiplicity (including both the N_c and C_F terms) and the LO one for three running coupling prescriptions. For comparison, we also show the results for $\bar{\alpha}_s(k_\perp)$ when including only the N_c terms (same as the curve "$\bar{\alpha}_s(k_\perp)$" in the left panel). For both figures $\sqrt{s} = 500$ GeV, $\eta = 3.2$ and the evolution of the color dipoles is obtained by solving the Balitsky-Kovchegov equation with the smallest dipole prescription using an MV [13] initial condition at $X_0 = 0.01$.

dominated by large values $x_\perp \sim 1/Q_s$ and thus it is independent of its lower limit $r_\perp$ in the approximation of interest. So long as the coupling $\bar{\alpha}_s \equiv \alpha_s N_c/\pi$ is independent of $r_\perp$, the final Fourier transform yields a vanishing result. On the contrary, if the coupling is chosen to depend upon the parent dipole size — as is the case for the two running coupling prescriptions most commonly used when solving the BK equation: the smallest dipole prescription and the Balitsky prescription [12] —, this dependence will lead to a large and unphysical contribution from the region at $x_\perp \gg r_\perp$, i.e. from soft (small $p_\perp$) primary gluons.

Because of this, and the fact that the BK equation can be written using the same integrals J and J_v, one could wonder why similar issues don't appear when solving it with these prescriptions. The reason is that the BK equation involves the difference between J and J_v, i.e.

$$\mathcal{J}(\boldsymbol{k}, \xi = 1) - \mathcal{J}_v(\boldsymbol{k}, \xi = 1) = \int \mathrm{d}^2\boldsymbol{r}\, \bar{\alpha}_s(r_\perp)\, \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}} \int \frac{\mathrm{d}^2\boldsymbol{x}}{(2\pi)^2} \frac{\boldsymbol{r}^2}{\boldsymbol{x}^2(\boldsymbol{x}+\boldsymbol{r})^2} \left[S(-\boldsymbol{x})S(\boldsymbol{r}+\boldsymbol{x}) - S(\boldsymbol{r})\right]. \quad (6)$$

Thus the spurious contributions coming from the large daughter dipoles region cancel in this case.

Based on this, we expect that using the daughter dipole prescription in the impact factor should lead to physical results: since $\bar{\alpha}_s(x_\perp)$ does not depend on $\boldsymbol{r}$, the integral (5) still vanishes after taking the Fourier transform when $x_\perp \gg r_\perp$. On the other hand, in the contributing region $x_\perp \sim r_\perp$, one recovers the prescription $\bar{\alpha}_s(r_\perp)$. To illustrate this we show in Fig. 1 (A) the results for the NLO/LO ratio obtained with different prescriptions for the running coupling used in the impact factor. We observe that the results obtained with the daughter dipole prescription are close to the ones obtained with a momentum space prescription $\bar{\alpha}_s(k_\perp)$ or with a fixed coupling. With the daughter dipole prescription it becomes possible to use the same coupling in the whole calculation, and, as in the case of a fixed coupling, there is no ambiguity between the "subtracted" and "unsubtracted" [6] formulations of the cross-section. Note, however, that it is not very natural to use the daughter dipole prescription when solving the BK equation since one generally expects that the scale of the running coupling should be set by the hardest scale in the problem.

So far we considered only the NLO corrections to the cross-section proportional to N_c. Another source of such corrections is proportional to C_F. These terms are affected by the same large daughter dipoles problem as the N_c terms, but an additional complication appears here. Indeed, the C_F terms contain collinear divergences which have to be absorbed into the DGLAP evolution of the parton distribution functions and fragmentation functions. Because this subtraction is performed in momentum space, it is not possible to rewrite all the C_F terms as double integrals over $\boldsymbol{r}$ and $\boldsymbol{x}$. Therefore, we cannot use the daughter dipole prescription for these terms. In addition, while with a fixed or momentum space running coupling the C_F terms vanish when $\xi \to 1$, this is no longer the case when the coupling depends on transverse coordinates, and this generates a spurious longitudinal logarithm. We thus consider that the most physical choice for these terms

is the momentum space prescription $\bar{\alpha}_s(k_\perp)$. In Fig. 1 (B) we show the results we obtain when including both the N_c and C_F NLO corrections with fixed, momentum and coordinate space running coupling. For comparison we also show the results obtained with the momentum space prescription including only the N_c NLO terms. This allows us to see that the inclusion of the C_F terms has a sizable effect and, being opposite in sign compared to the N_c terms, they reduce the size of the NLO corrections to the cross-section.

3. Conclusions

In this work we have identified the origin of the troublesome results obtained with the coordinate space running coupling prescription used in [7] in the calculation of forward particle production at next-to-leading order. This problem is due to the fact that the Fourier transform and the choice of the running coupling prescription do not commute, and that without a careful choice of this prescription some unphysical contributions no longer cancel after the final Fourier transform. We proposed [11] to overcome this problem by using the daughter dipole prescription for the N_c terms, as this preserves the required cancellations and leads to very similar results compared to a momentum scale choice. However, the same choice cannot be made for the C_F terms due to the subtraction of the collinear divergences, and the choice $\bar{\alpha}_s(k_\perp)$ seems to be mandatory for these terms.

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